Optimization Problems

- Solve applied minimum and maximum problems.
Find 2 positive #s where the product is 185 & the sum is a min.

\[ x \cdot y = 185 \]

\[ S = x + y \]

primary eq.

\[ y = \frac{185}{x} \]

\[ S = x + \frac{185}{x} \]

\[ S = x + 185 \cdot x^{-1} \]

\[ \frac{dS}{dx} = 1 - 185 \cdot x^{-2} \]

\[ \frac{dy}{dx} = 1 - \frac{185}{x} \]

1. \[ \frac{1}{x^2} = 0 \]

1. \[ \frac{185}{x^2} = 1 \]

\[ x^2 = 185 \]

\[ x = \pm \sqrt{185} \]

\[ x = \sqrt{185} \]

\[ \sqrt{185} \cdot y = 185 \]

\[ y = \sqrt{185} \]

\[ S'(x) = \frac{dy}{dx} \]

\[ S'(x) = - \frac{185}{x^2} \]

\[ x = \sqrt{185} \]

\[ y = \sqrt{185} \]

func is ↓

func is ↑
One of the most common applications of calculus involves the determination of minimum and maximum values.

Example 1 – Finding Maximum Volume

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches, as shown in Figure 3.53. What dimensions will produce a box with maximum volume?

\[ V = l \cdot w \cdot h \]
\[ V = x \cdot x \cdot h \]
\[ V = x^2 h \]

Primary eq. \[108 = x^2 + 4xh\]

\[ 108 - x^2 = 4xh \]
\[ h = \frac{108 - x^2}{4x} \]
\[ h = \frac{27x^2 - \frac{1}{4}x}{x} \]

\[ V = x^2 \left( 27x - \frac{1}{4}x \right) \]
\[ V = 27x - \frac{1}{4}x^3 \]

\[ \frac{dV}{dx} = 27 - \frac{3}{4}x^2 \]

0 = 27 - \frac{3}{4}x^2
\[ x^2 = \frac{36}{4} \]
\[ x = \pm6 \]
\[ x = 6 \]

108 = 36 + 24h
72 = 24h
h = 3

6in \times 6in \times 3in

v'(1) < 0, v'(6) < 0
func is ↓
func is ↓
GUIDELINES FOR SOLVING APPLIED MINIMUM AND MAXIMUM PROBLEMS

1. Identify all given quantities and all quantities to be determined. If possible, make a sketch.

2. Write a primary equation for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the back cover.)

3. Reduce the primary equation to one having a single independent variable. This may involve the use of secondary equations relating the independent variables of the primary equation.

4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
EXAMPLE 3 Finding Minimum Area

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1½ inches, and the margins on the left and right are to be 1 inch (see Figure 3.56). What should the dimensions of the page be so that the least amount of paper is used?

\[ xy = 24 \]
\[ A = (x + 3)(y + 2) \quad \text{primary eq.} \]
\[ y = \frac{24}{x} \]
\[ A = (x + 3)\left(\frac{24}{x} + 2\right) \]
\[ A = 24 + 2x + \frac{72}{x} + 6 \]
\[ A = 30 + 2x + \frac{72}{x} \]
\[ \frac{dA}{dx} = 2 - \frac{72}{x^2} \]
\[ \frac{2}{x^2} = 72 \]
\[ x^2 = 36 \]
\[ x = \pm 6 \]
\[ x = 6 \]

\[ \frac{d^2A}{dx^2} = 144x^{-3} \]
\[ = 144 \frac{1}{x^3} \]
\[ A''(6) = \text{pos} \quad \text{concave up} \quad \text{min} \]